Pattern distribution in faro words and permutations

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The faro shuffle [2, 4] is a well known technique to shuffle a deck of cards, where the deck is split in two at the middle, and the cards from the two halves are interlaced by taking alternatively the bottoms of packets. For two k-ary words u and v such that $0 \leq |u| - |v| \leq 1$, the faro shuffle of u and v is the k-ary word of length $|u| + |v|$ obtained by interlacing the letters of u and v as follows: $u_1v_1u_2v_2u_3v_3\ldots$. Let $\mathcal{S}_{n,k}$ be the set of n-length faro shuffles of two non-decreasing k-ary words. For instance, we have $\mathcal{S}_{4,2} = \{1111, 1112, 1121, 1122, 1212, 1222, 2121, 2122, 2222\}$. Since $\mathcal{S}_{n,k}$ is also the set of k-ary length n words w satisfying $w_i \leq w_{i+2}$ for $1 \leq i \leq n-2$, its cardinality is $\binom{\lfloor \frac{n}{2} \rfloor + k - 1}{k-1} \cdot \binom{\lceil \frac{n}{2} \rceil + k - 1}{k-1}$. Denote by $\mathcal{P}_n$ the set of permutations in $\mathcal{S}_{n,n}$ which we call faro permutations. Obviously, we have $\mathcal{P}_n \subset Av_n(321)$, $\mathcal{P}_3 = \{123, 132, 213\}$ and $|\mathcal{P}_n| = \binom{n}{\lfloor \frac{n}{2} \rfloor}$.

In this poster, we focus on the distribution and the popularity [1, 3] of consecutive patterns of length at most three in faro words and permutations. Our method consists in showing how patterns are getting transferred from faro words (or permutations) to lattice paths, which settles us in a more suitable ground in order to provide generating functions for the distribution and the popularity. We present a constructive bijection f between $\mathcal{S}_{n,k}$ and the set of dispersed Dyck paths of length $n + 2k - 2$ having $k - 1$ peaks. For any consecutive pattern of length at most three σ, we show how f transports the statistic of the number of occurrences of σ, and we derive enumerating results about its distribution on $\mathcal{S}_{n,k}$. Among them, we give the trivariate generating function enumerating words in $\mathcal{S}_{n,k}$ with respect to the length n, the arity k, and the number of descents. A counterpart study for faro permutations in $\mathcal{P}_n$ is also presented. Finally, open questions are presented and discussed.

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References

[1] M. Bóna. Surprising symmetries in objects counted by Catalan numbers. *The Electronic J. of Comb.*, 19(1)(2012), P62.

[2] P. Diaconis, R.L. Graham, and W.M. Kantor. The mathematics of perfect shuffles. *Advances in Applied Mathematics*, 4(1983), 175-196.

[3] S. Kitaev. *Patterns in Permutations and Words*. Springer, 2011.

[4] S.B. Morris. The basic mathematics of the faro shuffle. *Pi Mu Epsilon J.*, 6(1975), 85-92.