Packing coloring and subsets preserving path distance

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A packing coloring of a graph $G = (V, E)$ is a function $V \to \mathbb{N}$ such that the distance between any two nodes colored by the same color i is always greater than i. We note by $\chi_\rho(G)$ the corresponding minimal number of colors. Denote by T_2 the infinite binary tree. Sloper [1] proved that $\chi_\rho(T_2) \leqslant 7$. We show that $\chi_\rho(T_2) = 7$.

Another variant of packing coloring appears when we authorise only colors greater than k. In this case the minimal number of needed colors is denoted by $\chi_\rho^k(G)$. Let P_∞ denote the infinite path. Goddard et al. [2] proved that $\chi_\rho^k(P_\infty) \leqslant 3k + 2$.

A graph is asymptotically packing colorable if $\chi_\rho^k(G) < \infty$ for any k. It follows from [3] that hexagonal and rectangular infinite lattices are not asymptotically packing colorable. We also show that T_2 is not asymptotically packing colorable.

In order to characterise a family of asymptotically packing colorable graphs we introduce a following notion: a subgraph H of G *preserves a path distance* if there exists a bijection ϕ from the set of nodes of P_∞ to the set of nodes of H such that $d_{P_\infty}(u, v) \leqslant d_G\big(\phi(u), \phi(v)\big)$. Using this tool we prove that if graph G can be decomposed into a finite number of subgraphs while preserving a path distance, then G is asymptotically packing colorable. We also conjecture that the converse holds. It also turns out that the minimal number of subgraphs in a path distance preserving decomposition of G together with Goddard et al.'s result (discussed below) can be used to bound the parameter $\chi_\rho(G)$.

References

[1] C. Sloper, An eccentric coloring of trees, *The Australasian Journal of Combinatorics* **29**:309–321, 2004.

[2] W. Goddard, S. M. Hedetniemi, S. T. Hedetniemi, J. M. Harris, D. F. Rall, Broadcast chromatic numbers of graphs, *Ars Combinatoria* **86**:33-49, 2008.

[3] N. Gastineau, H. Kheddouci, O. Togni, Subdivision into i-packings and S-packing chromatic number of some lattices, *Ars Mathematica Contemporanea* **9**:331–354, 2015.